

Valuation for the Strategic Management of Research and Development Projects: The Deferral Option

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Abstract: The most widely used financial technique for evaluating projects is discounted cash flow; however, discounted cash flow analysis fails to consider flexibility. Real options analysis offers an alternative technique that provides value for the managerial flexibility that is inherent in most R&D projects. This article investigates the deferral option using computer simulation. There are five variables that determine the value of the deferral option, and simulations analyze these variables over a wide range of conditions. Sensitivity analysis on the five variables is performed and the results are discussed.

Keywords: Real Options, Project Valuation, Deferral Option, Net Present Value, Research and Development

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Research and development (R&D) projects are routinely evaluated to determine if the projects are feasible and worthy of continued funding. Most R&D organizations have more ideas than they have resources to fund them, so projects must compete for available resources, including money and talent. A widely used technique for evaluating projects is discounted cash flow (DCF). In this method, the net present

value (NPV) is determined by discounting forecasted future cash flows by a required rate of return. Despite its wide use, discounted cash flow biases evaluators toward conservative conclusions (Copeland and Antikarov, 2001). Good ideas are sometimes not pursued because the method provides an NPV that is often too low. Management usually has flexibility during the course of R&D projects, and this flexibility is not accounted for in the discounted cash flow technique.

Projects with NPVs that are high are considered good investments from the discounted cash flow perspective. Projects with NPVs that are negative are generally abandoned because they will not deliver the required return. Projects with NPVs close to zero require significant additional effort to determine if such projects should be funded or abandoned. Real options analysis can be used to add insight to the funding decision, especially when DCF analysis finds an NPV that is close to zero. Real options analysis offers an alternative that determines a value for managerial flexibility and provides an expanded net present value (ENPV).

NPV analysis is used to determine funding decisions for capital property. In the case of equipment purchases, uncertainty is low because prices can be obtained ahead of time from suppliers. In the case of funding under certainty, discounted cash flow analysis works extremely well. Under conditions of uncertainty, real options analysis may be preferred.

Decision tree analysis is another technique that is widely discussed in the literature. This method identifies all of the

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available decisions and creates a diagram to map the available options. Probability-weighted NPVs are determined for each decision tree. Decision tree analysis is closely linked with real options analysis; decision trees can be used to help identify various options that the project may contain.

To date, real options analysis has not been widely adopted in industry. One likely reason is that many managers are uninformed about this technique (Teach, 2003). Many studies have addressed the topic of project valuation. More than a dozen books have been written on the subject during the last 10 years on real options analysis alone, and additional books are being published at an ever-increasing rate. Many more than 100 journal articles that have been written on the subject, and real options analysis has become a fairly popular topic.

The difficulty with the published literature lies in the fact that most publications do not address the topic for direct application. The early books and many of the journal articles focus on the mathematical derivations of the calculations, focusing on Ito calculus and differential equations (Dixit and Pindyck, 1994; Trigeorgis, 1996; Bellalah, 2001, for example). Most practitioners find this overwhelming. Some of the recent books deal with the mathematics very lightly or not at all, which fails to provide the industrial practitioners with the necessary tools. Even the best of the books (and there are excellent books available) are so long that a person needs to digest hundreds of pages before they are able to attempt a calculation. The topic of real options is complex, and the mathematics is cumbersome. Modern computer software can aid in performing the calculations, but analysts still need to understand the process. Most of the published literature does not make the subject easy to apply.

There are five primary management options regarding R&D projects. First, a project may be delayed if future information will decrease the decision risks (the deferral option). Second, projects can be abandoned if their salvage value exceeds the project's future returns (the abandonment option). Projects may be expanded at a later date if market share exceeds expectations (the expansion option), or can be reduced in size if sales volumes fall short of forecasts (the contraction option). Finally, many projects occur in several phases, with each phase dependent on the success of a previous one (the sequential compound option). All of these options involve five variables: the future cash flows, the cost of implementation, the time horizon under consideration, the risk-free cost of money, and the volatility of the future returns.

The volatility is perhaps the most difficult of all of the variables to forecast. Volatility is defined as the standard deviation of the project's rate of return. Merck & Company first uses a volatility of 40%, and then performs the analysis again at a value of 60% (Nichols, 1994). Volatility is difficult to measure, and the option value is highly dependent on the volatility estimate. Volatility is, therefore, an important variable, and it is helpful to understand the sensitivity of an option value to the volatility of its underlying asset. Volatility estimation is described in the literature (Cobb and Charnes, 2004; Copeland and Antikarov, 2001; Herath and Park, 2002).

Sensitivity analysis can be performed on real options in the same manner as is done with financial options. There are several variables used in financial analysis that are used to determine the sensitivity of an option. These are known as Delta, Gamma, Vega, Rho, Theta, and Xi, and are collectively known as "The Greeks" (Deacon and Faseruk, 2000).

The intent of this study is to investigate methods of valuating research and development projects using the deferral option. The investigation identifies how to calculate a deferral option and identifies how the value of the option will change as the input conditions are varied over a wide range. The analysis will compare the relationships of future cash flow, investment costs, interest rates, time, and volatility with the estimated NPV of the project using computer simulations. Such valuation analysis can aid the firm in managing R&D projects for maximum strategic value. While options analysis can be used to evaluate intellectual property, this area was excluded from the current study. Intellectual property valuation is an area for further research.

Background

Projects are often implemented as action plans to support the goals and objectives of the firm (Gray and Larson, 2003). Every project should add value in accordance with the organization's strategic plan. Of course, some projects will be implemented for non-financial reasons. These include projects to correct safety problems, projects to improve product or process quality, and projects that are reactions to changes in market conditions. In order to discuss the valuation of projects, a basic understanding of valuation techniques is required. There are several techniques, and each has its appropriate use.

The valuation of new business opportunities depends on both the capabilities of the firm and the business strategy that is used. Strategic use of intellectual capital provides the strengths for sustainable competitive advantage of the firm. A new opportunity will not see commercialization unless business strategy and tactics are taken into account. In evaluating research and development projects, two issues must be addressed: (1) how the new knowledge will bring value to the firm (strategically, not numerically), and (2) quantifying the amount of value that the asset will provide (Davis and Harrison, 2001).

There are three accepted valuation methods used in accounting: market, cost, and income (Smith and Parr, 2000). The *market approach* is the most direct and easily understood of the methods. It simply uses the market to judge the value of a given good. There are only two requirements: an active public market (buyers and sellers), and an exchange of comparable properties. This is a basic method used widely for the valuation of nearly anything that is publicly sold in quantity. This method is not often used for the valuation of R&D projects or for intellectual property, because the basic requirements are not readily met. A market valuation cannot be done on a project (or an intellectual asset) when only one exists.

The *cost approach* deals with the replacement cost of a good. The subject property is given a value equal to the cost that it took to create or to replace the property. In the case of R&D projects, this is not a valid approach. The value of the project is expected to far exceed the cost it took to create it; therefore, the cost approach fails to provide a good measure of R&D project value.

The *income approach* focuses on the income-producing capability of the project. Value can be defined as the present value of future benefits to be derived by the owner of the project; therefore, the valuation process needs to quantify the future benefits, and discount them to their present value. In financial terms, the value of an asset can be measured by the present worth of the net economic benefit that can be achieved over the lifetime of the asset. For our purpose, the worth of the project is equal to

what the project can earn. The income approach is the method that is best suited for assessing the value of an R&D project.

At the heart of the income approach is the discounted cash flow technique. This involves the determination of the NPV of future cash flows by discounting the cash flows by a required rate of return. The discounted cash flow method is widely used to determine the value of projects, and has been embraced by industry. The required rate of return is typically the weighted average cost of capital of the firm (the effective interest rate that the firm must pay). Some firms impose a high interest rate as a hedge against risk, requiring high rates of return for high-risk projects. This higher interest rate is commonly called the Hurdle Rate (Meredith and Mantel, 2003).

Another method of determining the value of a project involves the use of real options. In general, the discounted cash flow method tends to be too conservative; good ideas are often not pursued because the method provides an NPV that is too low. The primary reason for this is the assumption that once the decision is made to fund a project, expenses and cash inflows occur without the possibility of being changed. In reality, management has the option of making changes a number of times during the life of the project, especially during the early stages (Miller and Park, 2002).

Real options analysis has received widespread attention and acclaim within academia since the early 1990s. Even though very few companies have extensive experience with real options, Copeland (2001) feels that in ten years real options will replace NPV as the central method for investment decisions.

Examples of real options include licenses for oil exploration, an option to purchase electricity at a set price at a future date, or an option to purchase land. Real options represent rights that are expected to be exercised later after more information becomes available about the value of that economic right. Real options, therefore, help in decision making under uncertain conditions. Judy Lewent, Chief Financial Officer of Merck, has said:

“When you make an initial investment in a research project, you are paying an entry fee for a right, but you are not obligated to continue that research at a later stage. Merck’s experience with R&D has given us a database of information that allows us to value the risk or volatility of our research projects, a key piece of information in option analysis. Therefore, if I use option theory to analyze that investment, I have a tool to examine uncertainty and to value it. ... To me, all kinds of business decisions are options” (Nichols, 1994).

Other recent examples of real options applications include the biotech industry (Borissiouk and Pell, 2001; Kellogg and Charnes, 2000).

Copeland (2001) has developed a four-step process to describe the actions needed to properly carry out a real options analysis. The steps include:

1. Compute the present value without flexibility using standard Discounted Cash Flow valuation.
2. Model the uncertainty using event trees. This helps build an understanding of how the present value develops over time. A DCF analysis of the resulting event tree should yield the same result as Step 1.

3. Identify and incorporate managerial flexibilities. By analyzing the event tree from Step 2, management options are identified, such as the option to defer the project to a later date.
4. Conduct real options analysis. Once the event tree is identified with the known options, the computational analysis may be carried out. If uncertainty is zero, the present value is the same as in Step 1. If uncertainty is significant and management has the ability to be flexible, the added option value can be significant.

Method

The first step in any option analysis is to identify the NPV, based on Equation 1.

$$NPV = -I_0 + \sum_{T=1}^T \frac{FV_T}{(1+r)^T} \quad (1)$$

where I_0 is the original investment
 FV_T are the future cash flows
 r is the interest rate
 T is the time increment

Once the NPV has been calculated, the flexibility of a project can be determined.

An R&D project can be treated as an option. Management can choose to fund, abandon, delay, or expand a project. R&D projects can, therefore, be structured as real options. The value of the real option, and the value of the project in total, can be calculated in a similar way as financial options are calculated. There are two primary tools used: the binomial option pricing model and the Black-Scholes pricing model. This work uses computer simulations to determine project values using both techniques.

A project must first be structured with the real options identified. This requires that the project be viewed in a strategic context, with barriers and options highlighted. In the case of a deferral option, the issue becomes one of waiting until more information is known before investing. The deferral option identifies the value of keeping the project funded while deferring the decision to implement or execute the project.

As an example, let us imagine a small consumer products company that is developing a new product. The company is not yet sure that the product will be economically viable, and has been performing a financial feasibility study. The present value of the future cash flows has been estimated to be approximately \$10 million, but the volatility of the market is fairly high. The investment cost of this opportunity will be \$10 million, one year in the future. The option to defer (or delay) the project to a time when more is known has value.

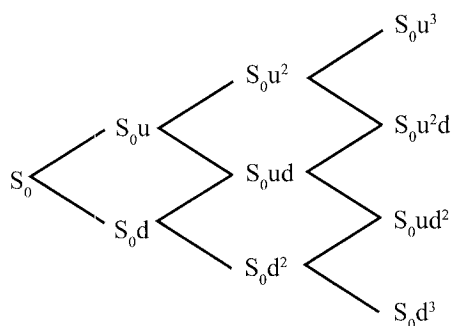
In discounted cash flow analysis, the present value of the investment cost would be subtracted from the present value of the future cash flows. In real options analysis, the option to invest in the project is simply an option—it is not an obligation. If the project is viewed as a “European Option,” the option can be exercised only at the end of the time frame. When valued as an “American Option,” it is assumed that the option can be exercised any time during the time frame. The option creates an expanded net present value (ENPV), which can be calculated:

$$ENPV = NPV + \text{Option Value} \quad (2)$$

When NPV is quite large, the option value will not have a significant impact on the decision: the NPV signals that the project is worthy of investment. When NPV is very negative, even the best option values will not be large enough to value it as a profitable project, and the project should not be pursued. If the future cash flows are known with certainty, then the discounted cash flow technique should be used. Real options have their best use under conditions of uncertainty, and where management has the ability and the willingness to exercise its flexibility. The option value places a price on the worth of this flexibility, and the ENPV identifies how much the firm should be willing to pay to keep the project (or option) open.

Binomial Lattices. The binomial options approach uses a lattice to demonstrate alternative possibilities over time (Dixit and Pindyck, 1994). The lattice may be used for valuating both real and financial options. The starting point is the present value of the future cash flows. Over time T, two conditions can result: one up and one down (hence the term binomial). More detailed lattices can be made to illustrate either more time or simply more steps in time. Exhibit 1 shows a binomial lattice with three time steps.

Exhibit 1. Binomial Lattice



The solution can be obtained using one of two approaches. Financial options often use a market-replicating portfolio to solve the binomial problem. Real options generally use a risk-neutral probability approach. The two approaches are directly related and will yield the same answer if structured correctly. This article works exclusively with risk-neutral probabilities.

Using the risk-neutral probability approach, each time-step may be calculated (Mun, 2002). The up-step is defined as

$$u = e^{\sigma \sqrt{\delta t}} \quad (3)$$

where σ is the volatility of the cash flows
 δt is the length of each time-step

The down-step is defined as

$$d = e^{-\sigma \sqrt{\delta t}} = 1/u \quad (4)$$

The risk-neutral probability is defined as

$$p = \frac{e^{r\delta t} - d}{u - d} \quad (5)$$

where r is the risk-free interest rate.

Each type of real option requires a calculation in a slightly different way, but the solution always forms at least two lattices. This example demonstrates the valuation of a deferral option. The first lattice, illustrated in Exhibit 2, shows the evolution of the underlying project. For our example above, let us assume the following:

- $S_0 = \$10$ million PV of the future cash flows
- $X = \$10$ million project cost
- $T = 1$ year
- $N = 3$ time steps
- $\delta t = 0.33$ year (T/N)
- $\sigma = 30\%$ volatility
- $r = 5\%$ (Treasury rate for a 3-year bond)

$$NPV = 10 - \frac{10}{(1.05)^1} = 0.48$$

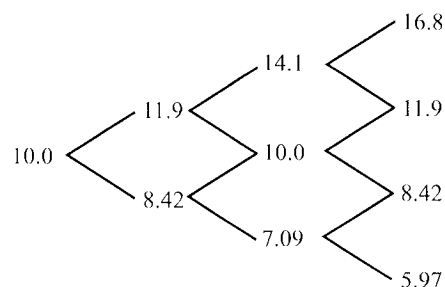
$$u = e^{\sigma \sqrt{\delta t}} = e^{0.30 \sqrt{0.33}} = 1.19$$

$$d = 1/u = 0.842$$

$$p = \frac{e^{r\delta t} - d}{u - d} = \frac{e^{0.05(0.33)} - 0.842}{1.190 - 0.842} = 0.505$$

If the present value of the future cash flows, S_0 is \$10 million, and $u = 1.19$, then the first up position is S_0u or $(10)(1.19) = 11.9$. The down position is S_0d or $(10)(0.842) = 8.42$. This procedure is continued until the lattice is complete.

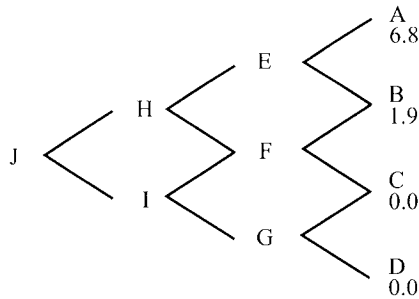
Exhibit 2. Lattice of the Underlying Asset



The second lattice, shown in Exhibit 3, is the option valuation lattice. Calculations start on the right side of the lattice, identifying the value of exercising the option at that point in time. The option value at time T is first calculated at each right hand node; it is the evolved underlying asset value minus the cost. If the value minus the cost is less than zero, then the project will not be executed, and the resulting value is zero. The second lattice will not have any points less than zero, and the option value as a result will never be less than zero. In the top position A, the value is S_0u^3 (which is equal to 16.8) minus the cost X (which is 10.0). Point A is, therefore, valued at \$6.8 million. Under these conditions, the project should be implemented because it results in a positive cash flow. This same procedure is continued down the column, and node B is valued at 1.9. Again, the project should be implemented. At node C, the value is the greater of $[S_0ud^2 - X]$ or zero. Since $[S_0ud^2 - X]$ is equal to -1.58 , the node is valued at zero, and the project would not be implemented under these

conditions. This represents management flexibility, which the DCF approach does not consider. The ability to not execute the project under unfavorable conditions has value. Node D is valued at $\text{MAX}(S_0 d^3 - X, 0.0)$, so it also has a value equal to zero, and the project would again be abandoned.

Exhibit 3. Lattice of the Expanded NPV



Internal points on the lattice are calculated using a method known as backward induction (Mun, 2002). The point is determined based on the probabilities of achieving the points already calculated on its right, discounted for the time period δt . The discounting is traditionally performed assuming continuous compounding. The calculation for determining point E is then

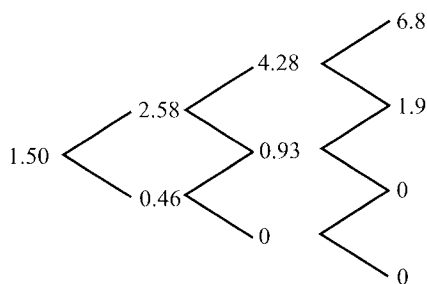
$$\begin{aligned} & [(P)(6.8) + (1-P)(1.9)] e^{-\delta t} \\ & = [(0.505)(6.8) + (0.495)(1.9)] e^{-(0.05)(0.33)} = 4.28 \end{aligned}$$

This process is continued until the lattice is complete. At the extreme left side, the final value is the option value. As shown in Exhibit 4, this value is 1.50. The value of keeping the project open up to one year is:

$$\text{ENPV} = \text{NPV} + \text{Option value} = 0.48 + 1.5 = \$2.0 \text{ million}$$

During this time, additional information can be gathered. It is also worth spending up to \$2 million during this time to keep the option open.

Exhibit 4. Complete Option Lattice



A call option on non-dividend paying stocks will be worth more if it is held open than if the call is executed (Bodie et al., 2002). In the example above, it was noted that the value of point E is 4.28. This is the value if the option is held open. The value at this same point if the option were exercised would be the underlying asset price less the cost, or $14.1 - 10.0 = 4.1$. This is an example of how the binomial lattice can be used to compare the value of holding the option open rather than exercising it. The lattice also

demonstrates that the deferral option (with no dividends) will always be worth more alive than dead. In real options terms, there is no advantage to executing the project early, assuming that there is no cost of waiting.

While the binomial lattice can be easily understood once a person has a little experience with it, it is extremely cumbersome to calculate. Computer software is now available to calculate binomial lattices, which makes the procedure much faster. The Real Options Analysis Toolkit by Decisioneering, Inc. is used to calculate a variety of real option methods. This software is a spreadsheet-based (Excel) application that calculates real option values, expanded NPV, and identifies values at each point in a lattice. The Real Options Analysis Toolkit was used to perform the binomial lattice calculations used in this article.

Black-Scholes. The Black-Scholes equation has been used for a number of years to determine the value of financial options (Bodie et al., 2002). The equation approximates the value of a European call option, based on the current stock price (S_0), strike price (X), volatility (σ), risk-free interest rate (r), and time to expiration (T). A call option is a financial instrument that gives the holder the option to buy an asset at a specified price on or before some expiration date. A put option gives the holder the option to sell an asset at a specified price. The equation is (Black and Scholes, 1973):

$$C = S_0 N(d_1) - X e^{-rT} N(d_2) \quad (6)$$

$$\begin{aligned} \text{where } d_1 &= \frac{\left(\ln \frac{S_0}{X} \right) + \left(r + \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}} \\ d_2 &= d_1 - \sigma \sqrt{T} \end{aligned}$$

$N(d_x)$ is the cumulative standard normal distribution of the variable d_x

For the previous example, we can calculate the option value using the Black-Scholes equation,

$$d_1 = \frac{\left(\ln \frac{10.0}{10.0} \right) + \left(0.05 + \frac{0.3^2}{2} \right) T}{0.3 \sqrt{1}} = 0.3167$$

$$d_2 = 0.3167 - 0.3 \sqrt{1} = 0.0167$$

$$C = (10.0) (0.624) - (10.0) e^{-(0.05)(1)} (0.507) = 1.42$$

The above result of 1.42 can be compared with the binomial lattice result of 1.50. A derivation of the above equation is also available for determining the value of a put option, as based on the put-call parity theorem (Gibson, 1991).

$$P = X e^{-rT} [1 - N(d_2)] - S [1 - N(d_1)] \quad (7)$$

The binomial lattice and the Black-Scholes equation will provide similar, but not identical, results. The Black-Scholes model is a continuous function, while the binomial lattice is a

discrete function (and we used only three time steps). The five primary variables involved in the Black-Scholes calculation for financial assets can be directly related to real assets. These are shown in Exhibit 5 (Trigeorgis, 1996).

Exhibit 5. Option Variables

Variable	Black-Scholes	Real Options
T	Time to expiration	Time to expiration
r	Risk-free interest rate	Risk-free interest rate
X	Exercise price	Implementation cost
S	Stock price	PV of future cash flows
σ	Volatility of stock returns	Volatility of project returns

Sensitivity Analysis. Sensitivity analysis can be used in conjunction with real options. There are several sensitivity models widely used with financial options, known as the Greeks (Deacon and Faseruk, 1999). These are partial differential equations, based on the Black-Scholes model. The Greeks define changes in option value relative to changes in each independent variable. For instance, delta is defined as the change in option value for each incremental change in the value of the underlying asset (S). Vega is the change in option value due to changes in volatility. Several of these tools are used extensively in tracking financial options, but there has been limited published research on their use with real options.

The Deferral Option

Changes in the Underlying Asset. The value of the deferral option will vary with the changes in the input parameters, and can be calculated using the computer software previously mentioned. One of the most critical parameters is the value of the underlying asset. In the case of financial options, this is the price of the underlying stock. In the case of real options, this is the present value of the future cash flows of the project. Exhibit 6 shows the changes in the option value as the value of the underlying asset changes, based on the binomial lattice method. The four separate lines represent four different project costs, ranging from 10 to 200. For this graph, the volatility is held constant at 50%, the risk free interest rate is held constant at 5%, and the time frame is constant at five years with five time-steps. The shape of the curves follows those of financial call options. The deferral option is a form of a call option, and follows the same mathematics.

If the Black-Scholes pricing model were used to calculate the option values, similar (but not identical) results would be obtained. Exhibit 7 illustrates the option value using the Black-Scholes equation. Exhibit 8 shows some of this information in more detail, demonstrating several characteristics when the cost X is equal to 100. The dashed curve shows the call value according to the Black-Scholes pricing model, while the superimposed triangles show the value when the binomial lattice is used for the same conditions. These have very similar values. The dotted line shows the call value when time is equal to one year instead of five years.

Exhibit 6. Option Value with Changes in Cash Flow; $\sigma = 0.50$, $r = 0.05$, $T = 5$

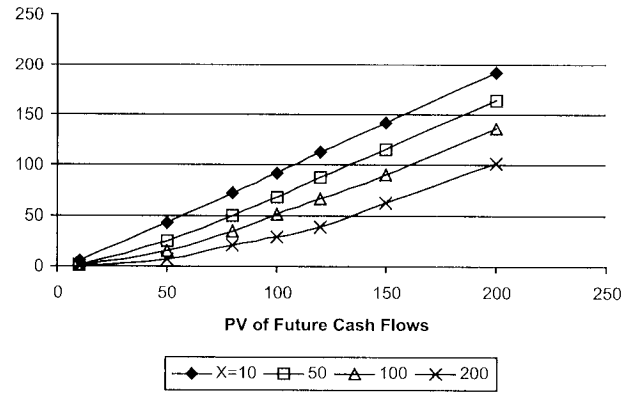


Exhibit 7. Option Value Using the Black-Scholes Model; $\sigma = 0.50$, $r = 0.05$, $T = 5$

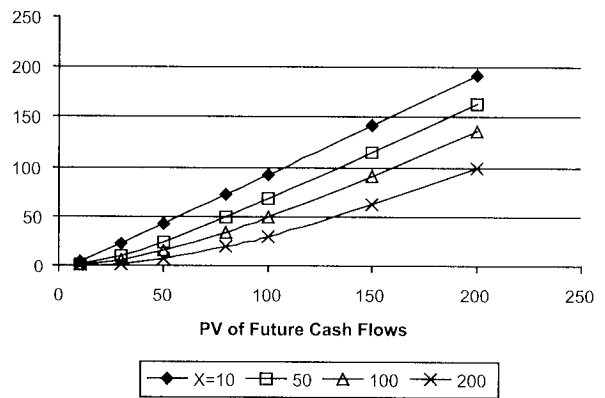
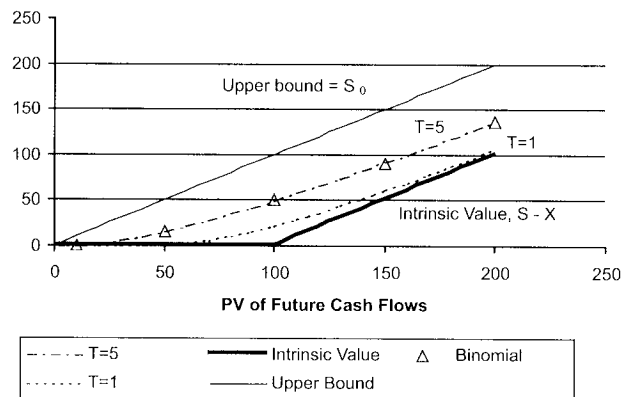


Exhibit 8. Option Value when $X = 100$, $\sigma = 0.50$, $r = 0.05$



All options have two types of value: the intrinsic value and the time value. The lower line in Exhibit 8 shows the intrinsic value, $S - X$, for the real option. This is the payoff that could be obtained if the option were immediately exercised.

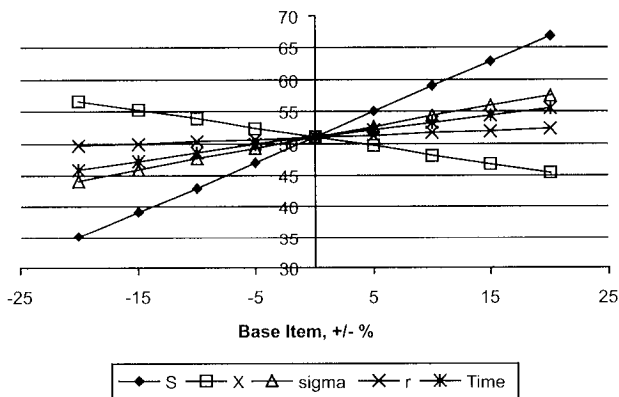
The "time value" is the difference between the call option line and the intrinsic value line. It is the portion of the option value that can be attributed to the fact that the option still has time to expiration (Bodie et al., 2002). The time value is always greatest where the present value of the future cash flows (S) is equal to the cost (X). Notice that the size of the time value is much greater when

$T = 5$ compared to $T = 1$. Real options will have a larger time value than most financial options, because the time frame is usually greater with real options.

The maximum value of a deferral option will be the present value of the future cash flows, S_0 . This is shown as the upper boundary line in Exhibit 8. The minimum value is zero; the option value will never be less than zero because the option can be allowed to expire without being exercised (you don't have to fund the project). As S increases, the minimum value becomes $[S - PV(X)]$, or the present value of the future cash flows minus the present value of the future project cost.

Sensitivity. Numerous authors have described a standard sensitivity method, sometimes known as a Spiderplot, that compares a dependent variable to multiple attributes (Park, 2002; Eschenbach, 2003). This tool is a convenient way of showing the relative sensitivity of a parameter to a number of variables. Exhibit 9 shows a spiderplot that demonstrates the sensitivity of the deferral option value to the five independent variables. The option value is most sensitive to changes in S , the present value of the future cash flows. In calculating the option value, estimates of the future income are the most crucial, and should be predicted with great care. The next most important variable is sigma, the volatility of the future cash flows. The least critical variable is the interest rate.

Exhibit 9. Sensitivity of the Option (center point at $S = 100, X = 100, \sigma = 0.50, r = 0.05, T = 5$)



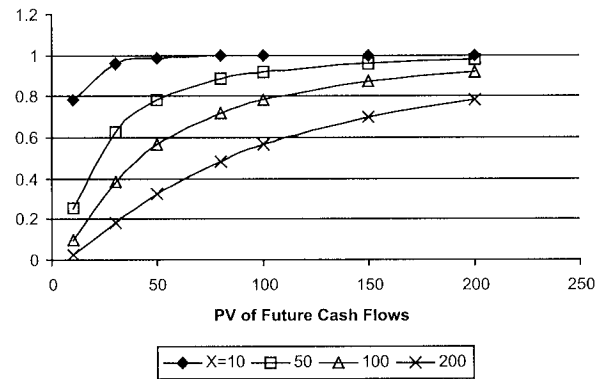
The Greek sensitivity parameter for the project cash flows is known as Delta. It is defined as the change in option value for each unit change in the underlying asset S (the present value of future cash flows). For a deferral option, Delta is defined as:

$$\text{Call Delta} = \partial C / \partial S = N(d_1) \quad (8)$$

This relationship is also known as the hedge ratio, and represents the slope of the curve in Exhibit 7 at the given point. Exhibit 10 shows delta as calculated from Equation 8. Essentially the same result can be obtained using the binomial lattice and calculating $\Delta C / \Delta S$. The value of Delta increases as the value of the underlying project increases, with a maximum value of 1.0 and a minimum value of zero. The fact that Delta is always equal to or greater than zero confirms the relationship between the option value and the future cash flows; increased cash flows will yield a larger option value. The volatility, interest rate, and time are all held constant. An example of determining the sensitivity of the option value

can be done using Exhibit 10. The value of Delta is 0.78 when $X = 50$ and $S = 50$. This means that for every unit increase in S , the option value will increase by 0.78.

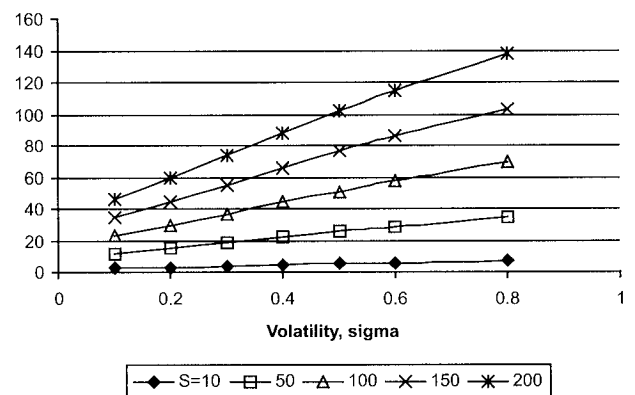
Exhibit 10. Call Delta; $\sigma = 0.50, r = 0.05, T = 5$



Volatility. The second most important variable in estimating the option value is the volatility of the future cash flows. Volatility is perhaps the most difficult of all of the variables to estimate, especially in an R&D scenario. The volatility is defined as the standard deviation of the project's rate of return.

Exhibit 11 shows the relationship of the deferral option value to changes in volatility when the asset value S equals the project cost X . This graph is based on results from the binomial lattice. Option values increase with increases in volatility. This is because the probability of the upside potential increases as the variability increases. The probability of the downside potential does not increase since the minimum value of the option is zero. Exhibit 12 shows the volatility relationship based on the Black-Scholes equation under the same conditions. It can be seen that Exhibit 11 is very similar to Exhibit 12. The option values that are calculated by the binomial lattice are similar to those calculated using the Black-Scholes pricing model.

Exhibit 11. Volatility; $X = S, r = .05, T = 5$

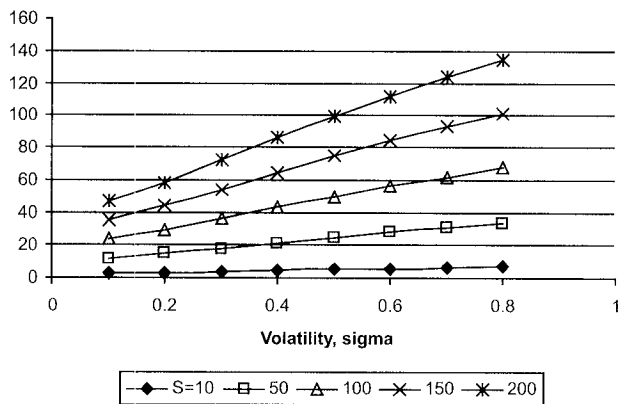


The change in the option value per unit change in the volatility is known as the Greek term Vega. As shown in Exhibit 13, Vega reaches a maximum at about $\sigma = 30\%$. Exhibit 13 shows Vega based on the equation:

$$\text{Vega} = \partial C / \partial \sigma = S\sqrt{T} N'(d_1) \quad (9)$$

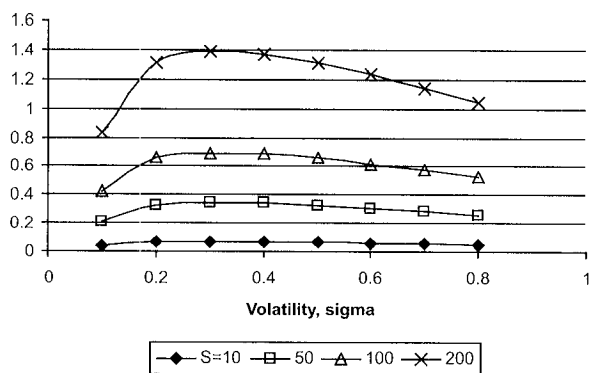
where $N'(d_1) = \frac{e^{-\frac{1}{2}(d_1)^2}}{\sqrt{2\pi}}$

Exhibit 12. Volatility Based on Black-Scholes



Vega will always be greater than or equal to zero. An increase in the volatility will result in an increased option value and a decrease in volatility will decrease the option value.

Exhibit 13. Vega, Based on Partial Differentials, X = S, r = 0.05, T = 5



It can be seen that volatility can have a profound effect on the option value. When Vega is 0.8, a 10% change in volatility will cause an 8% change in the option value. Because volatility is very difficult to estimate, it is important to understand how sensitive the option value will be to changes in volatility.

Project Cost. The option value is highly dependent on the cost of the project. When forecasting, it is important to have an accurate and reliable cost that is required to implement the project. Exhibit 14 shows the nature of how the option value will change with changes in the project cost, based on the binomial lattice. The option value decreases as X increases at all values of S, reaching a minimum value of zero. This is expected because increasing costs of the project will decrease the overall project value.

The change in the option value per unit change in the project cost is sometimes referred to as the Greek Xi, and is defined mathematically as the partial differential $\partial C / \partial X$. Exhibit 15 shows this sensitivity relationship, based on the partial differential of the Black-Scholes equation:

Call $\Xi = \partial C / \partial X = e^{-rt} N(d_2)$ (10)

The sensitivity of the option value to the project cost is not dependent on the future cash flows, S.

Exhibit 14. Project Cost Based on Black-Scholes

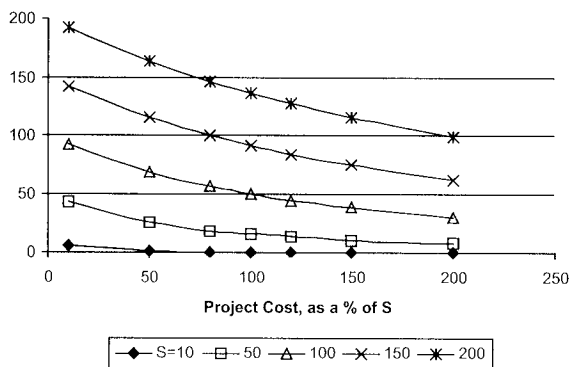
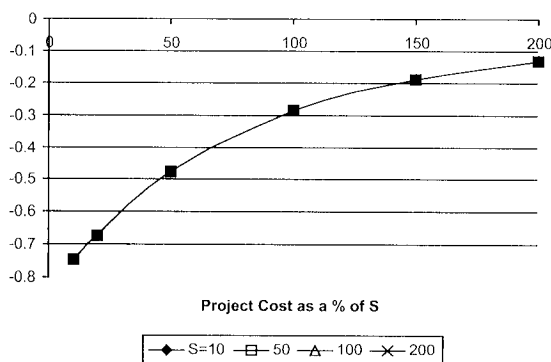


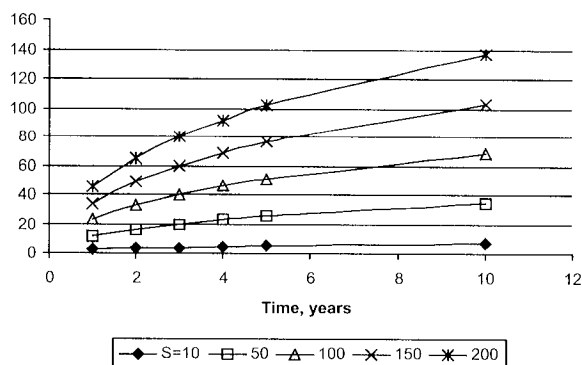
Exhibit 15. Xi, Based on Partial Differentials; $\sigma = 0.50, r = 0.05, T = 5$



Time to Maturity. If a project is being considered for funding sometime in the next five years, then the time to maturity is five years. A five-year timeline means that the interest rate must be an annualized rate good for five years and the volatility must be based on an annualized standard deviation.

The deferral option value increases with increases in the time horizon. The option increases in value because the chances of ending with a positive value increase with time, while the chances of ending with a negative value do not (the option will never be worth less than zero). The relationship is shown in Exhibit 16.

Exhibit 16. Time Relation; X = S, $\sigma = 0.50, r = 0.05$

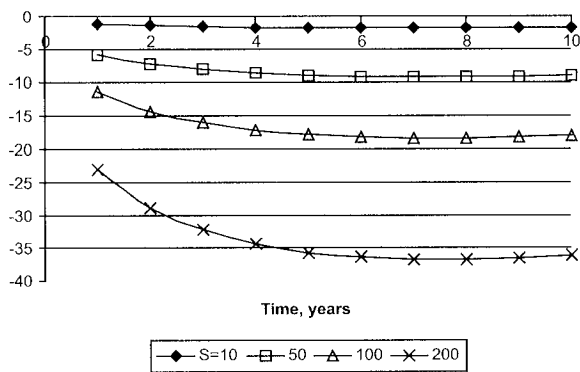


The sensitivity function for time is known as Theta, and is defined as a negative correlation as follows:

$$\text{Call Theta} = -\partial C / \partial T = -\left(\frac{S\sigma}{2\sqrt{T}}\right) N'(d_1) - rXe^{-rT} N(d_2) \quad (11)$$

Theta, based on Equation 11, is shown in Exhibit 17. Theta is defined as negative because as time passes the option becomes less valuable. As the time to maturity decreases, Theta increases and becomes closer to zero.

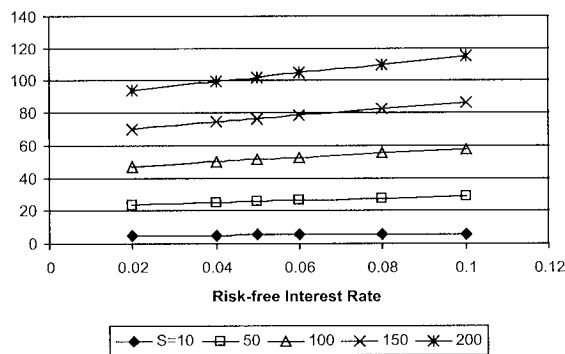
Exhibit 17. Theta, Based on Partial Differentials; $X = S, \sigma = 0.50, r = 0.05$



Interest Rate. In discounted cash flow, the interest rate is often increased to compensate for risk. Unfortunately, risk and interest rates are difficult to correlate with any accuracy. In real options analysis, risk is transferred to the volatility function. The interest rate used in real options is, therefore, a risk-free rate based on the time horizon. If the project has a timeline of five years, then choose the rate for five-year Treasury bonds. If the project has an option timeline different from five years, use a corresponding Treasury rate.

The option value increases with increasing interest rates, as shown in Exhibit 18. The project costs are discounted to the present, and higher interest rates will decrease the present value of the future costs.

Exhibit 18. Risk-free Interest Rate; $X = S, \sigma = 0.50, r = 0.05$



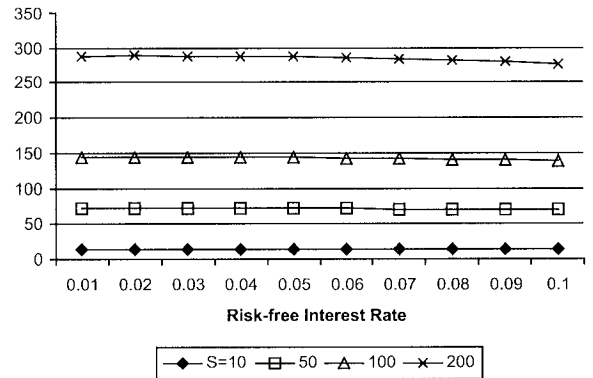
The sensitivity function for the interest rate is known as Rho, and is defined by

$$\text{Call Rho} = \partial C / \partial r = Txe^{-rT} N(d_2) \quad (12)$$

The continuous function for Rho, based on Equation 12, is shown in Exhibit 19. In this graph, the interest rate is a fraction, so a 1% change in the interest rate when $S = 100$ will give a

change in the call value of approximately $(0.01)(144) = 1.44$. Rho is always positive for a call option, demonstrating the fact that the call option price is positively correlated with the interest rate. Sensitivity to the interest rate is relatively minor compared to the effect of the volatility and the cost.

Exhibit 19. Rho Based on Partial Differentials; $X = S, \sigma = 0.50, T = 5$



Dividends. In financial options, stock dividends need to be considered. Dividends are a distribution of the corporation's wealth, and thereby decrease the value of the underlying stock. In real options, there is a corollary. While there is value in waiting for additional information, there is also risk. There is potential lost revenue by not being in the market, other companies may market a similar product first, or there may be hidden costs of not funding the project. The deferral option needs to consider the cost of not making a decision.

The format for calculating the risk of waiting is to treat the risk as a dividend. The potential cost of waiting needs to be estimated, and is then divided by the present value of the future cash flows:

$$D = \text{Dividend rate} = \text{The cost of waiting} / S_0 \quad (13)$$

Let us take our original example and include a cost of waiting. In the example, we had a present value of future cash flows of \$10 million, and a project cost of \$10 million. Let us assume that after waiting 4 months there is an estimated risk of waiting worth \$0.5 million. Let us further assume that this same \$0.5 million risk occurs at every decision point, occurring every four months. The binomial lattice from Exhibit 2 will be altered, as shown in Exhibit 20.

The annual dividend rate will be $(0.5)(3)/10.0 = 0.15$ or 15%. Because of this dividend, the value of the underlying lattice will be significantly lower, reducing the value of the option. The option lattice can be solved as described earlier. Without the dividend, the option was worth \$1.50 million. With the dividend, the option is worth only \$0.72 million, a decrease of 52%.

In the Black-Scholes model, dividends can be accounted for as shown in Equation 14 (Bodie et al., 2002).

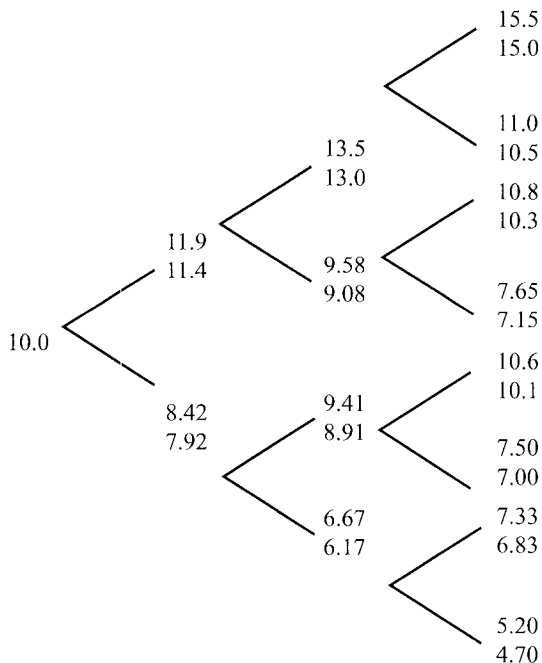
$$C = S_0 e^{-DT} N(d_1) - Xe^{-rT} N(d_2) \quad (14)$$

where D = dividend rate, and

$$d_1 = \frac{\left(\ln \frac{S_0 e^{DT}}{X} \right) + \left(R + \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

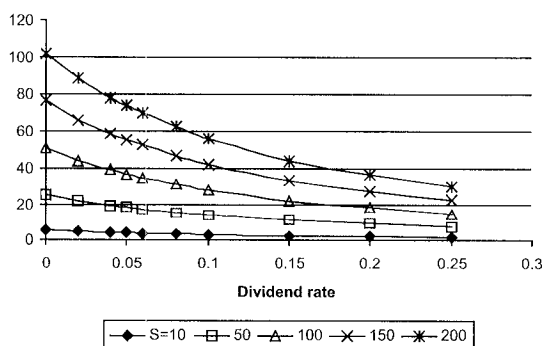
Exhibit 20. Lattice of the Underlying Asset with Dividends



For our example, the option value with dividends is 0.62 when calculated by the modified Black-Scholes method, compared to 0.72 when calculated using the binomial lattice.

The relationship of the dividend to the option value is shown in Exhibit 21. The option value decreases significantly as the dividend rate increases. The higher the cost of waiting, the lower the value the deferral option will have.

Exhibit 21. The Cost of Waiting, $S = X, \sigma = 0.50, r = 0.05, T = 5$



Sensitivity Using Monte Carlo Analysis. The problem with determining the sensitivity of the option value to each of the option parameters is that it looks at each variable in isolation. In reality, these inputs interact. Changes in the call option value due to changes in the input variables, with several variables changing

at the same time, can be determined by performing multiple simulations. This can be carried out with Monte Carlo analysis using Crystal Ball® software.

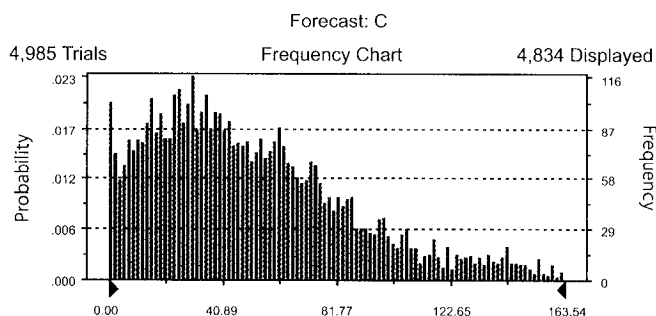
A simple example is created using the variables shown in Exhibit 22. The mean present value of the future cash flows will be given as \$100, with the cost at time T also \$100. The volatility will be assumed to be 0.5; the interest rate set at 5%, and the time set at five years. These are the same conditions as were used to create the sensitivity example for Exhibit 9. In this current example, all values will be allowed to vary at the same time, with the standard deviations identified in Exhibit 22. The Black-Scholes model assumes that S will follow a lognormal distribution. It will also be assumed that the volatility, cost, time, and interest rate will all follow a normal distribution; however, values less than zero will not be allowed and will be truncated from the distribution.

Exhibit 22. Monte Carlo Variables

Variable	Mean Value	Standard Deviation	Distribution
S, PV of cash flows	100	50.0	Lognormal
σ , Volatility	0.50	0.25	Normal, ≥ 0
X, Cost	100	40.0	Normal, ≥ 0
T, Time	5 years	0.5	Normal
r, Interest rate	0.05	0.01	Normal

The Black-Scholes equation is used to calculate the value of a call option using the data in Exhibit 22. Five thousand trials are performed. The range of option values that we can expect to obtain is illustrated in Exhibit 23, and the statistical results are shown in Exhibit 24. The resulting distribution of the option value is similar to the lognormal distribution of the future cash flows, which demonstrates the influence that S has on the option value. The mean value of the distribution is 54.76. The Black-Scholes equation will provide a call option value from the above variables of 49.60. The higher value from the Monte Carlo analysis is because the distribution is skewed to the right, slightly increasing the mean value.

Exhibit 23. Option Value Distribution, Black-Scholes Call Option



Sensitivity charts are available in Crystal Ball that illustrate the impact of each of the input variables on the value of the option. The first sensitivity chart, shown in Exhibit 25, is based on rank correlation coefficients. Correlation coefficients provide a measure of the degree to which assumptions and forecasts

change together. Positive coefficients show that an increase in the assumption is associated with an increase in the forecast, while negative coefficients indicate a negative correlation. The larger the value of the correlation coefficient, the stronger is the relationship (Evans and Olson, 2002).

Exhibit 24. Option Value Results

Statistic	Value
Mean	54.76
Median	45.63
Mode	---
Standard Deviation	42.06
Variance	1,769.45
Skewness	1.36
Kurtosis	5.35
Coefficient of Variability	0.77
Range Minimum	0.00
Range Maximum	267.13
Range Width	267.13
Mean Standard Error	0.60

Exhibit 25. Sensitivity Chart Measured by Rank Correlation

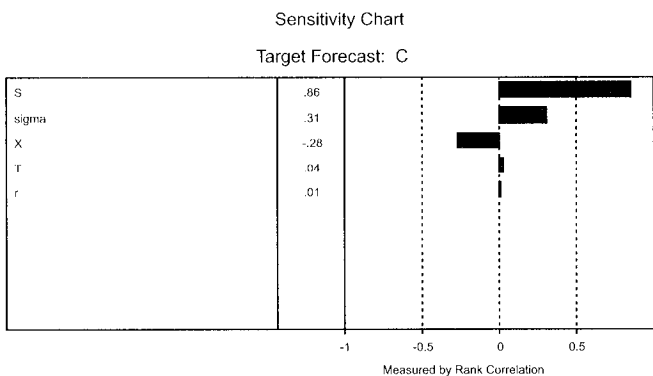


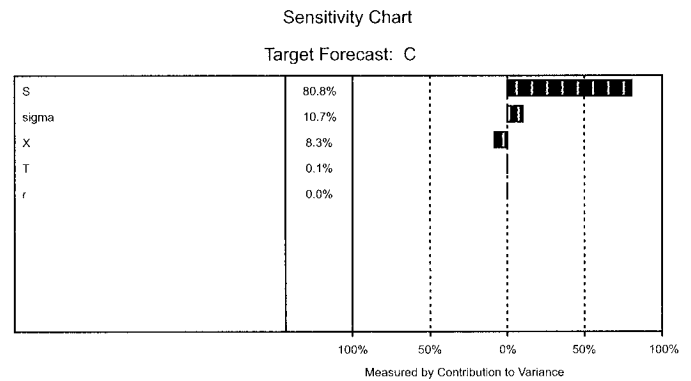
Exhibit 25 demonstrates that the present value of the future cash flows exerts the greatest influence on the option, followed by the volatility and then the cost for our set of assumptions. Increases in S and volatility will increase the option value. Increases in cost decrease the option value. The time horizon and the rate of return have much less influence.

Exhibit 26 illustrates the sensitivities as a percent of the contribution to the variance of the option value. This identifies the percentage of the variance of the option value that is due to a specific input variable. This chart demonstrates that 80.8% of the variance in the option value is due to the variance of the present value of the future cash flows. Volatility and cost are also major factors contributing to variance of the option value, while the time horizon and the rate of return are not.

The sensitivity charts will change with any change in the above input variables, so these results should not be considered universal. Also, Monte Carlo results are determined from a series

of trials, or simulations, and each series is unique. Small variations will occur each time the simulation is repeated; however, Monte Carlo analysis can be used to determine the sensitivity of the option value to changes in the input variables, when all of the inputs are in flux.

Exhibit 26. Sensitivity Chart Measured by Contribution to Variance



Implications for the Engineering Manager

The valuation of a project is an aspect of project management that can be crucial to the success of a project. Valuation is discussed extensively in the academic literature and in the popular business press. The issue is relevant to business accounting and finance, and valuation is an important part of tax law. Discounted cash flow is widely used in industry, and the engineering manager needs to be aware of the problems that these methods present. Discounted cash flow undervalues many projects.

This problem can be overcome by using real options analysis; however, options analysis has been criticized for being a "black box." Many managers do not understand the methods and do not understand how a given option value is calculated. Option valuation can be approached in a similar way as a process: there are five input variables and one output variable. The output is, therefore, dependent on each of the inputs, and the response of the output is unique to each of the input variables. This approach adds insight to how the option value changes as any of its inputs change. Real option values are determined based on a set of forecasts, including future cash flows and future costs. By their very nature, the inputs are inexact, and will create an option value that is no more precise than its inputs. In determining the characteristic of an option, it is, therefore, important to understand not only what variables change, but also what the sensitivity of the option is to the inaccuracy of the inputs.

The binomial lattice approach is a very flexible technique that is based on simple mathematics. The method is fairly easy to learn, and can be understood by a wide variety of interested parties (the accounting department, management, etc.). The binomial lattice can also be coupled with Monte Carlo analysis. The merging of these methods creates a tool that is very flexible yet provides a complete picture of the resulting project value. Any or all of the inputs can be changed with respect to their mean, their standard deviation, and their distribution. Likewise, the project value is completely determined, including its mean, standard deviation, and distribution. The basic tools are readily available: an ordinary personal computer using Microsoft Excel and Crystal Ball software are all that is required.

Conclusions

The deferral option can provide a more accurate project value. First, the NPV of the project is determined. Next, the problem must be structured as an option. The value of this option is dependent on five variables:

- The present value of the sum of future cash flows
- The cost of implementing the project
- The volatility of the future cash flows
- The timeline until the option is exercised
- The risk-free interest rate.

This differs from performing traditional discounted cash flow in that the option provides a value for the managerial flexibility inherent in the project. In the deferral option, the cost of waiting may need to be integrated into the project value. Waiting can be treated like dividends, and incorporated into the option value math.

Given our base condition ($S = 100$, $X = 100$, $\sigma = 50\%$, $r = 5\%$, and $T = 5$ years), a 20% increase in each of the variables (determined independently) will provide a change in the option value as follows:

PV of future cash flows	+31.1 %
Volatility	+13.6 %
Cost	- 11.1 %
Timeline	+10.0 %
Interest rate	+ 2.7 %

The above sensitivities show the effect on option value as a single variable changes. In reality, there are interactions among the variables. Interest rates are based on the time horizon. Volatility must be based on the appropriate time increment. Interest rate increases will increase the option value because of the discounting effect on the project cost. Many of the variables are interdependent.

The following relationships have been illustrated below and summarized in Exhibit 27.

- The option value as it relates to the future cash flows
- The sensitivity of the option value to its input variables when one variable changes
- Call Delta, the sensitivity of the option value to changes in the future cash flows
- The option value as it relates to the volatility
- Vega, the sensitivity of the option value to changes in volatility

Exhibit 27. Summary of Relationships

Increases in this Variable	Effect on the Option Value	Effect on the Sensitivity of the Option Value
Future cash flows, S	Increase	Increase
Volatility, σ	Increase	Increase to a maximum then decrease
Cost, X	Decrease	Increase
Time horizon, T	Increase	Decrease
Interest rate, r	Increase	Decrease slightly
Dividend, D (cost of waiting)	Decrease	n/a

- The option value as it relates to the project cost
- Call Xi, the sensitivity of the option value to changes in project cost
- The option value as it relates to the time until implementation
- Call Theta, the sensitivity of the option value to changes in time
- The option value as it relates to the risk-free interest rate
- Call Rho, the sensitivity of the option value to changes in the interest rate
- The cost of waiting
- The sensitivity of the option value as all input variables change simultaneously.

The identified sensitivities provide a priority for guiding forecasting. The present value of future cash flows and their volatility should be forecasted with great care, while the interest rate can be estimated with somewhat less precision. The cost of waiting also needs to be considered, as this can be a significant factor in determining the value of deferring a decision.

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